

Superconductivity in Uranium Compounds

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On the basis of microscopic theory it is demonstrated how the coupling between the electrons by means of magnetization fluctuations in ferromagnetic metal with orthorhombic symmetry gives rise equal spin pairing superconducting state with general form of the order parameter dictated by symmetry. The strong upturn of the upper critical field along b -direction above 5 Tesla in UCoGe is explained by the increase of pairing interaction caused by the suppression of the Curie temperature by magnetic field parallel to b -axis. It is proposed that similar phenomenon at much higher field must take place also for the field directed along the magnetically hardest a -direction.

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I. INTRODUCTION

Superconductivity arising in ferromagnet state of uranium compounds UGe₂, URhGe and UCoGe possessing extremely high upper critical field and the re-entrant superconductivity in large external magnetic field is obviously related to triplet type superconductivity (for the most recent reviews see¹⁻³). These ferromagnets have orthorhombic structure with magnetic moments oriented along one of the crystallographic axis (a -axis in UGe₂, c -axis in URhGe and UCoGe). The general form of the order parameters in orthorhombic ferromagnet superconductors with triplet pairing allowed by symmetry has been given in Ref.4. In neglect of interband pairing these superconducting states are reduced to the equal spin pairing states.^{5,6}

It is quite natural to consider this type of superconductivity as magnetically mediated. Here, however, there are two possibilities distinguished by the nature of ferromagnetic state. Either it is due to spin-fluctuation exchange in itinerant electron ferromagnet, or due to electron interaction with spin waves in system of ferromagnetically ordered localized magnetic moments.

Recently there was presented an argumentation^{7,8} that ferromagnetism in uranium compounds is mostly supported by the moments located at uranium atoms. This case the most plausible pairing mechanism is due to interaction between the conduction electrons by means of spin waves in the system of localized moments. Such type model applied to the ferromagnetic superconductor URhGe⁹ successfully explains the reentrant superconductivity phenomenon in this material.

Even earlier there was developed a microscopic description of triplet superconductivity in ferromagnetic materials based on pairing interaction derived from phenomenological spectrum of fluctuations in the orthorhombic ferromagnet with strong magnetic anisotropy.¹⁰ This approach allows to establish the interplay between pressure dependence of the Curie temperature and the critical temperature of superconducting transition. Also, there was demonstrated that depending of an external field orientation parallel or perpendicular to the direction of spontaneous magnetization the effective amplitude of

pairing interaction proves to be decreasing or increasing function of magnetic field that allows to explain the drastic difference in magnitudes of upper critical field in these directions.

Here we continue and develop further the approach of Ref.10. There will be presented more exact derivation of the magnetic susceptibility tensor and the amplitudes of the pairing interaction. The treatment of superconducting state will be done taking into account both paramagnetic and orbital interaction of magnetic field with electron spins and charges. We shall show how the coupling between the electrons by means of magnetization fluctuations in ferromagnetic metal with orthorhombic symmetry leads to formation of superconducting state with general form of the order parameter dictated by symmetry established in Ref.5,6. The role of orthorhombic anisotropy originating of spin-orbital interaction in formation of superconducting state will be demonstrated explicitly.

One of particular properties of UCoGe is the peculiar behavior of the upper critical field.¹¹ Namely, the upper critical field in b -direction reveals almost vertical upturn at fields around 5 tesla. Strong up-curvature is observed also in H_{c2} in a direction but at lower temperatures and higher magnetic fields. In addition, at low temperature $T = 0.09K$ which is much smaller than the zero-field superconducting temperature $T_{sc} = 0.6K$ a steep angular dependence of H_{c2} was reported when the field was slightly tilted from the a axis toward the c axis.¹¹ This steep angular dependence of H_{c2} cannot be explained at all with the anisotropic Ginzburg-Landau formula, even if the field dependence of effective mass m^* is taken into account.

The description of sharp angular dependence of the upper critical field was developed in frame of quite specific Ising type itinerant electron interaction approach.¹² Namely, the pairing interaction has been taken in form of wave vector-frequency dependent Fermi gas magnetic susceptibility. To stress the role of longitudinal fluctuations there was kept only z -component of susceptibility $\chi_{zz}(\mathbf{q}, \omega)$. The parallel to c axis external field dependence of susceptibility has been chosen in nonanalytic form quite different from known mean-field dependence.

There was considered single band model with pairing only between spin-up electrons. The H_{c2} angular dependence was obtained by solving the Eliashberg equations. The good correspondence of this description to the observations is not astonishing because it uses many baseless assumptions specially chosen to get correspondence with experiment.

Here we describe qualitatively the upper critical field temperature behavior as originating from pairing interaction field dependence. The strong upturn of the upper critical field along b -direction above 5 Tesla in UCoGe is explained by the increase of pairing interaction caused by the suppression of the Curie temperature by magnetic field parallel to b -axis. Similar mechanism also stimulates the arising of the reentrant superconducting state in URhGe in fields along b -axis about 12 Tesla discovered about decade ago¹³. It should be noted that the presented ideas about the relationship between ferromagnetic properties and superconductivity quite recently have been essentially supported experimentally¹⁴.

The paper is organized as follows. In Chapter II we introduce the microscopic Ginzburg-Landau-Gor'kov description of two-band equal-spin pairing superconductivity arising in orthorhombic metal due to electron pairing by means of exchange of magnetic fluctuations. Then in Chapter III we derive the susceptibility in orthorhombic ferromagnet that allows us to establish the field dependent pairing interaction. The structure of superconducting state is found in Chapter V. Finally, in Chapter VI we consider the field dependent effective critical temperature. The summary and discussion are contained in Conclusion.

II. GINZBURG - LANDAU - GOR'KOV EQUATIONS

We consider pairing originating from the attraction

$$H_{int} = -\frac{1}{2}\mu_B^2 I^2 \int d^3\mathbf{r} d^3\mathbf{r}' S_i(\mathbf{r}) \chi_{ij}(\mathbf{r} - \mathbf{r}') S_j(\mathbf{r}') \quad (1)$$

between electrons with magnetic moments μ_B by means electron-magnon interaction in ferromagnetic media with orthorhombic symmetry. Here, $\mathbf{S}(\mathbf{r}) = \psi_\alpha^\dagger(\mathbf{r}) \boldsymbol{\sigma}_{\alpha\beta} \psi_\beta(\mathbf{r})$ is the operator of electron spin density, $\chi_{ij}(\mathbf{r})$ is the media susceptibility, $I \approx \frac{T_c}{nm^2}$ is an exchange constant, T_c is the Curie temperature, n is the concentration of uranium atoms with zero-temperature magnetic moment m . The explicit form of pairing interaction due to electron-magnon coupling in weak coupling approximation is established in the same manner as it is done for electron-phonon coupling (see fi book¹⁵).

The BCS pairing interaction due to electron-magnon coupling derived¹⁶ from Eq.(1) is

$$H_{pairing} = \frac{1}{2} \sum_{\mathbf{k}\mathbf{k}'} V_{\alpha\beta\gamma\delta}(\mathbf{k}, \mathbf{k}') a_\alpha^\dagger(\mathbf{k}) a_\beta^\dagger(-\mathbf{k}) a_\gamma(-\mathbf{k}') a_\delta(\mathbf{k}'), \quad (2)$$

where

$$V_{\alpha\beta\gamma\delta}(\mathbf{k}, \mathbf{k}') = V_{ij}(\mathbf{k}, \mathbf{k}') (i\sigma_i \sigma_y)_{\alpha\beta} (i\sigma_j \sigma_y)_{\gamma\delta}^\dagger \quad (3)$$

and

$$V_{ij}(\mathbf{k}, \mathbf{k}') = -\mu_B^2 I^2 \left(\frac{1}{2} Tr \hat{\chi}^u(\mathbf{k}, \mathbf{k}') \delta_{ij} - \chi_{ij}^u(\mathbf{k}, \mathbf{k}') \right) \quad (4)$$

is expressed through the odd part of media static susceptibility $\hat{\chi}^u(\mathbf{k}, \mathbf{k}')$ found in the next section.

The upper critical field is determined as the eigenvalue of the linear equation for the order parameter

$$\Delta_{\alpha\beta}(\mathbf{k}, \mathbf{q}) = -T \sum_n \sum_{\mathbf{k}'} V_{\alpha\lambda\mu}(\mathbf{k}, \mathbf{k}') \times G_{\lambda\gamma}(\mathbf{k}', \omega_n) G_{\mu\delta}(-\mathbf{k}' + \mathbf{q}, -\omega_n) \Delta_{\gamma\delta}(\mathbf{k}', \mathbf{q}), \quad (5)$$

where the matrix of order parameter is

$$\hat{\Delta} = \begin{pmatrix} \Delta^\uparrow & \Delta^{\uparrow\downarrow} \\ \Delta^{\uparrow\downarrow} & \Delta^\downarrow \end{pmatrix}, \quad (6)$$

$G_{\lambda\gamma}(\mathbf{k}', \omega_n)$ is the matrix of normal metal Green function. In absence of external field and when the magnetic field is parallel to the spontaneous magnetization it is diagonal

$$\hat{G}_n = \begin{pmatrix} G^\uparrow & 0 \\ 0 & G^\downarrow \end{pmatrix}, \quad (7)$$

where

$$G^{\uparrow,\downarrow} = \frac{1}{i\omega_n - \xi_{\mathbf{k}}^{\uparrow,\downarrow}}. \quad (8)$$

For the external field \mathbf{H} oriented either in the (x, z) or in the (y, z) plane, where the coordinates x, y, z correspond to the a, b, c crystallographic directions, it is natural to choose the spin quantization axis along the direction of the total magnetic field $(h + H_z)\hat{z} + H_x\hat{x}$ (or $(h + H_z)\hat{z} + H_y\hat{y}$) and to introduce the corresponding shift of chemical potential in the spin-up, spin-down bands. Here h is the exchange field. This case, also one needs to write the interaction matrix in proper spin axis, as it is done in Ref.10. This makes the following treatment much more cumbersome. One can ignore this complicity in assumption that the exchange field is much larger than an external field used in measurement of superconducting properties in uranium ferromagnetic superconductors¹⁷. Under this assumption one can work with Eqs. (4), (7), (8). Abandonment of this assumption leads to the appearance of corrections of the order of $\sim H_x^2/h^2$ or $\sim H_y^2/h^2$ to all obtained results.

In view of large band splitting¹⁷ one may neglect the terms containing the products $G_1^\uparrow G_2^\downarrow + G_1^\downarrow G_2^\uparrow$. Then the equations for the spin-up Δ^\uparrow and the spin-down Δ^\downarrow order parameter components are

$$\Delta^\uparrow(\mathbf{k}, \mathbf{q}) = -T \sum_n \sum_{\mathbf{k}'} \{ V^{\uparrow\uparrow}(\mathbf{k}, \mathbf{k}') G^\uparrow(\mathbf{k}', \omega_n) G^\uparrow(-\mathbf{k}' + \mathbf{q}, -\omega_n) \Delta^\uparrow(\mathbf{k}', \mathbf{q}) + V^{\uparrow\downarrow}(\mathbf{k}, \mathbf{k}') G^\downarrow(\mathbf{k}', \omega_n) G^\downarrow(-\mathbf{k}' + \mathbf{q}, -\omega_n) \Delta^\downarrow(\mathbf{k}', \mathbf{q}) \}, \quad (9)$$

$$\Delta^\downarrow(\mathbf{k}, \mathbf{q}) = -T \sum_n \sum_{\mathbf{k}'} \{ V^{\downarrow\uparrow}(\mathbf{k}, \mathbf{k}') G^\uparrow(\mathbf{k}', \omega_n) G^\uparrow(-\mathbf{k}' + \mathbf{q}, -\omega_n) \Delta^\uparrow(\mathbf{k}', \mathbf{q}) + V^{\downarrow\downarrow}(\mathbf{k}, \mathbf{k}') G^\downarrow(\mathbf{k}', \omega_n) G^\downarrow(-\mathbf{k}' + \mathbf{q}, -\omega_n) \Delta^\downarrow(\mathbf{k}', \mathbf{q}) \}. \quad (10)$$

Here

$$V^{\uparrow\uparrow}(\mathbf{k}, \mathbf{k}') = V_{xx} + V_{yy} + iV_{xy} - iV_{yx} = -\mu_B^2 I^2 \chi_{zz}^u, \quad (11)$$

$$V^{\downarrow\downarrow}(\mathbf{k}, \mathbf{k}') = V_{xx} + V_{yy} - iV_{xy} + iV_{yx} = -\mu_B^2 I^2 \chi_{zz}^u, \quad (12)$$

$$V^{\uparrow\downarrow}(\mathbf{k}, \mathbf{k}') = -V_{xx} + V_{yy} + iV_{xy} + iV_{yx} = -\mu_B^2 I^2 (\chi_{xx}^u - \chi_{yy}^u - 2i\chi_{xy}^u), \quad (13)$$

$$V^{\downarrow\uparrow}(\mathbf{k}, \mathbf{k}') = -V_{xx} + V_{yy} - iV_{xy} - iV_{yx} = -\mu_B^2 I^2 (\chi_{xx}^u - \chi_{yy}^u + 2i\chi_{xy}^u). \quad (14)$$

One can see that the equations for Δ^\uparrow and for Δ^\downarrow are entangled. Hence, we deal with two-band superconducting state similar to A_2 state of superfluid ^3He . This property is supported by the recent low temperature thermal conductivity measurements under magnetic field¹⁸. It should be stressed that unlike to ^3He where entanglement between spin-up and spin-down components in absence of spin-orbit interaction is absent that leads to two subsequent phase transitions first to the spin-up and then to the spin-up-down superconducting states¹⁹ these components in orthorhombic ferromagnet interact each other already in linear approximation that gives rise one common phase transition to the A_2 type state. This is the particular property of superconducting state in an orthorhombic symmetry crystal with anisotropic susceptibility $\chi_{xx} \neq \chi_{yy}$. One band non-unitary superconducting state similar to $^3\text{He}-A_1$ is admissible in an isotropic metal with negligibly small spin-orbital coupling.

Let us now find the susceptibility.

III. MAGNETIC SUSCEPTIBILITY OF ORTHORHOMBIC FERROMAGNET

The expression for static magnetic susceptibility is obtained following phenomenological approach of Ref.10. The starting point is the Landau free energy of orthorhombic ferromagnet in magnetic field $\mathbf{H}(\mathbf{r})$

$$\mathcal{F} = \int dV (F_M + F_\nabla), \quad (15)$$

where in

$$F_M = \alpha_z M_z^2 + \alpha_y M_y^2 + \alpha_x M_x^2 + \beta_z M_z^4 + \beta_{yz} M_z^2 M_y^2 + \beta_{xz} M_z^2 M_x^2 - \mathbf{M}\mathbf{H}, \quad (16)$$

we bear in mind the orthorhombic anisotropy but, unlike to Ref.10, the density of gradient energy will be taken in

exchange approximation

$$F_\nabla = \gamma_{ij} \frac{\partial \mathbf{M}}{\partial x_i} \frac{\partial \mathbf{M}}{\partial x_j}. \quad (17)$$

Here, the x, y, z are directions of the spin axes pinned to a, b, c crystallographic directions correspondingly,

$$\alpha_z = \alpha_{z0}(T - T_{c0}), \quad (18)$$

$\alpha_x > 0$, $\alpha_y > 0$ and matrix γ_{ij} is

$$\gamma_{ij} = \begin{pmatrix} \gamma_{xx} & 0 & 0 \\ 0 & \gamma_{yy} & 0 \\ 0 & 0 & \gamma_{zz} \end{pmatrix}. \quad (19)$$

Let us take the magnetic field as the sum of the constant part and the coordinate dependent small addition

$$\mathbf{H}(\mathbf{r}) = (H_x + \delta H_x(\mathbf{r}))\hat{x} + (H_y + \delta H_y(\mathbf{r}))\hat{y} + (H_z + \delta H_z(\mathbf{r}))\hat{z}. \quad (20)$$

By variation of functional (15) in respect to the components of magnetization we arrive to equations

$$2\alpha_x M_x + 2\beta_{xz} M_z^2 M_x - \gamma_{ij} \frac{\partial^2 M_x}{\partial x_i \partial x_j} = H_x + \delta H_x, \quad (21)$$

$$2\alpha_y M_y + 2\beta_{yz} M_z^2 M_y - \gamma_{ij} \frac{\partial^2 M_y}{\partial x_i \partial x_j} = H_y + \delta H_y, \quad (22)$$

$$2\alpha_z M_z + 4\beta_z M_z^3 + 2\beta_{xz} M_z M_x^2 + 2\beta_{yz} M_z M_y^2 - \gamma_{ij} \frac{\partial^2 M_z}{\partial x_i \partial x_j} = H_z + \delta H_z \quad (23)$$

In constant magnetic field $\mathbf{H} = H_y \hat{y} + H_z \hat{z}$ the equilibrium magnetization projections are determined by equations:

$$M_y = \frac{H_y}{2(\alpha_y + \beta_{yz} M_z^2)}, \quad (24)$$

$$M_z^2 = \frac{1}{2\beta_z} \left(-\alpha_z - \frac{\beta_{yz} H_y^2}{4(\alpha_y + \beta_{yz} M_z^2)^2} \right) + \frac{H_z}{4\beta_z M_z}. \quad (25)$$

We see that under a magnetic field perpendicular to direction of spontaneous magnetization the Curie temperature decreases as

$$T_c = T_c(H_y) = T_{c0} - \frac{\beta_{xz}H_y^2}{4\alpha_y^2\alpha_{z0}}. \quad (26)$$

Thus, near the Curie temperature the z -component of magnetization is determined by the equation

$$M_z^2 \cong \frac{\alpha_{0z}}{2\beta_z}(T_c(H_y) - T) + \frac{H_z}{4\beta_z M_z}. \quad (27)$$

The first and the last term in this equation correspond to the spontaneous and the induced part of magnetization along z direction. It should be noted that the Curie temperature is the critical temperature of second order phase transition in ferromagnetic state only at $H_z = 0$. Otherwise it is a "temperature" of crossover from paramagnetic to ferromagnetic state under external field parallel to spontaneous magnetization.

One can obtain similar formulas for the constant magnetic field oriented in (x, z) plane $\mathbf{H} = H_x\hat{x} + H_z\hat{z}$. This case

$$M_x = \frac{H_x}{2(\alpha_x + \beta_{xz}M_z^2)}, \quad (28)$$

$$M_z^2 \cong \frac{\alpha_{0z}}{2\beta_z}(T_c(H_x) - T) + \frac{H_z}{4\beta_z M_z}, \quad (29)$$

$$T_c = T_c(H_x) = T_{c0} - \frac{\beta_{xz}H_x^2}{4\alpha_x^2\alpha_{z0}}. \quad (30)$$

Experimentally the Curie temperature is suppressed by magnetic field parallel to b crystallographic axis^{11,20}. In UCoGe the noticeable decrease is started at fields about 5 Tesla such that at fields about 15 Tesla the Curie temperature falls to zero.¹¹ On the contrary, for field in a -crystallographic direction a changes of T_c till the field about 15 Tesla have not been revealed as it could be expected for the magnetically hardest axis. One can speculate, however, that the Curie temperature $T_c(H_x)$ begins a decrease at fields higher than 20 Tesla.

Let us find first the susceptibility at finite field in (y, z) plane. Taking magnetization as the sum of the constant part and the coordinate dependent small addition

$$\mathbf{M}(\mathbf{r}) = M_y\hat{y} + M_z\hat{z} + \delta M_y(\mathbf{r}) + \delta M_z(\mathbf{r}), \quad (31)$$

one can obtain from the equations (22), (23) the linear response of magnetization to the coordinate dependent part of magnetic field. The corresponding Fourier components of magnetization are

$$\delta M_y(\mathbf{k}) = \chi_{yy}(\mathbf{k})\delta H_y(\mathbf{k}) + \chi_{yz}(\mathbf{k})\delta H_z(\mathbf{k}), \quad (32)$$

$$\delta M_z(\mathbf{k}) = \chi_{zy}(\mathbf{k})\delta H_y(\mathbf{k}) + \chi_{zz}(\mathbf{k})\delta H_z(\mathbf{k}), \quad (33)$$

where

$$\chi_{yy}(\mathbf{k}) = \frac{c}{D}, \quad \chi_{zz}(\mathbf{k}) = \frac{a}{D}, \quad (34)$$

$$\chi_{yz}(\mathbf{k}) = \chi_{zy}(\mathbf{k}) = -\frac{b}{D}, \quad (35)$$

$$a = 2\alpha_y + 2\beta_{yz}M_z^2 + \gamma_{ij}k_ik_j = \frac{H_y}{M_y} + \gamma_{ij}k_ik_j, \quad (36)$$

$$b = 4\beta_{yz}M_zM_y, \quad (37)$$

$$c = 8\beta_zM_z^2 + \frac{H_z}{2M_z} + \gamma_{ij}k_ik_j, \quad (38)$$

$$D = ac - b^2. \quad (39)$$

Like in the paper²¹ we shall use the following estimations for the constants in the GL free energy:

$$\beta_z \approx \frac{T_c}{2(m^2n)^2}, \quad \gamma_{xx} \approx \gamma_{yy} \approx \gamma_{zz} \approx \frac{T_cr^2}{m^2n}. \quad (40)$$

Corresponding GL expression for the z -component of magnetization at $H_z = 0$ is

$$M_z^2 = (mn)^2 \frac{T_c(H_y) - T}{T_c}. \quad (41)$$

Here n is the concentration of uranium atoms with zero-temperature magnetic moment m , r is the distance between nearest neighbor uranium atoms.

The expressions for the susceptibility component are valid for small wave vectors $k \ll k_F$. We shall expect, however, that they are still qualitatively valid at large wave vectors transfer $k \cong k_F$ determining pairing interaction. Then taking into account $k \cong k_F$ we come to estimation

$$\gamma_{ij}k_ik_j \approx \frac{T_c}{m^2n}. \quad (42)$$

Let us assume

$$ac \gg b^2 \quad (43)$$

that is certainly true when M_z is decreased together with the Curie temperature. Hence, one can work with the susceptibilities along c -axis and b axis in more simple form

$$\chi_{zz}(\mathbf{k}) \cong \frac{1}{c} = \frac{1}{8\beta_zM_z^2 + \frac{H_z}{2M_z} + \gamma_{ij}k_ik_j}, \quad (44)$$

$$\chi_{yy}(\mathbf{k}) \cong \frac{1}{a} = \frac{1}{\frac{H_y}{M_y} + \gamma_{ij}k_ik_j} = \frac{1}{\chi_y^{-1} + \gamma_{ij}k_ik_j}. \quad (45)$$

where the magnetization M_z is determined by Eq.(27) and $\chi_y = \frac{M_y}{H_y}$ is the constant part of susceptibility along a direction.

The same formula for $\chi_{zz}(\mathbf{k})$ is valid at finite field in (x, z) plane but this case the magnetization M_z is determined by Eq.(29). For x -component of susceptibility we have correspondingly

$$\chi_{xx}(\mathbf{k}) \cong \frac{1}{a} = \frac{1}{\chi_x^{-1} + \gamma_{ij}k_ik_j}. \quad (46)$$

The odd part of z -component of susceptibility is found as

$$\begin{aligned} \chi_{zz}^u(\mathbf{k}, \mathbf{k}') &= \frac{1}{2}[\chi_{zz}(\mathbf{k} - \mathbf{k}') - \chi_{zz}(\mathbf{k} + \mathbf{k}')] \\ &= \frac{2\gamma_{ij}k_ik'_j}{(8\beta_zM_z^2 + \frac{H_z}{2M_z} + \gamma_{ij}(k_ik_j + k'_ik'_j))^2 - (2\gamma_{ij}k_ik'_j)^2} \end{aligned} \quad (47)$$

The pairing interaction is determined by this formula at $\mathbf{k} = k_F \hat{k}$. In that follows we shall keep only angular dependence of interaction in numerator of Eq.(47) neglecting by angular dependence of the orthorhombic symmetry terms in denominator $\gamma_{ij}(k_i k_j + k'_i k'_j) \approx 2\gamma k_F^2$ as well by all higher harmonics of interaction determined by the last term in denominator. Hence, we obtain

$$\chi_{zz}^u(\mathbf{k}, \mathbf{k}') \cong \frac{2\gamma_{ij} k_i k'_j}{(8\beta_z M_z^2 + \frac{H_z}{2M_z} + 2\gamma k_F^2)^2}. \quad (48)$$

Found in similar manner the odd part of susceptibility x -component at finite field in (x, z) plane is

$$\chi_{xx}^u(\mathbf{k}, \mathbf{k}') \cong \frac{2\gamma_{ij} k_i k'_j}{(\chi_x^{-1} + 2\gamma k_F^2)^2}. \quad (49)$$

The odd part of susceptibility y -component at finite field in (y, z) plane is

$$\chi_{yy}^u(\mathbf{k}, \mathbf{k}') \cong \frac{2\gamma_{ij} k_i k'_j}{(\chi_y^{-1} + 2\gamma k_F^2)^2}. \quad (50)$$

Thus, we have found all the susceptibility components determining the pairing interaction besides $\chi_{xy}^u(\mathbf{k}, \mathbf{k}')$. The absence of off-diagonal susceptibility in orthorhombic crystal is direct consequence of exchange approximation we have used for the energy of magnetic inhomogeneity Eq.(17). Out of exchange approach the gradient energy in orthorhombic magnet contains also the invariants like

$$\gamma_{xy} \frac{\partial M_x}{\partial x} \frac{\partial M_y}{\partial y}. \quad (51)$$

This inevitably leads to appearance of off-diagonal components of susceptibility

$$\chi_{xy}^u(\mathbf{k}, \mathbf{k}') \cong -\frac{\gamma_{xy}(k_x k'_y + k'_x k_y)}{(\chi_x^{-1} + \gamma k_F^2)(\chi_y^{-1} + \gamma k_F^2)}. \quad (52)$$

IV. FIELD DEPENDENT PAIRING INTERACTION

For the pairing interaction Eq.(11) we obtain

$$\begin{aligned} V^{\uparrow\uparrow}(\mathbf{k}, \mathbf{k}') &= V^{\downarrow\downarrow}(\mathbf{k}, \mathbf{k}') \\ &= -\frac{\mu_B^2 I^2 \gamma_{ij} k_i k'_j}{2(\gamma k_F^2)^2 \left(\frac{4\beta_z M_z^2}{\gamma k_F^2} + \frac{H_z}{4M_z \gamma k_F^2} + 1 \right)^2} \end{aligned} \quad (53)$$

The angular dependent pre-factor in this equation can be rewritten as follows

$$\frac{\mu_B^2 I^2 \gamma_{ij} k_i k'_j}{2(\gamma k_F^2)^2} = \frac{T_c}{2n} \left(\frac{\mu_B}{m} \right)^2 \tilde{\gamma}_{ij} \hat{k}_i \hat{k}'_j, \quad (54)$$

where $\tilde{\gamma}_{xx} = \gamma_{xx}/\gamma, \dots$ are numbers of the order of unity and $\hat{k}_i = k_i/k_F$ are components of the unit vector \mathbf{k} . Thus, we obtain

$$\begin{aligned} V^{\uparrow\uparrow}(\mathbf{k}, \mathbf{k}') &= V^{\downarrow\downarrow}(\mathbf{k}, \mathbf{k}') = -V_1 \tilde{\gamma}_{ij} \hat{k}_i \hat{k}'_j \\ &= -V_1 (\tilde{\gamma}_{xx} \hat{k}_x \hat{k}'_x + \tilde{\gamma}_{yy} \hat{k}_y \hat{k}'_y + \tilde{\gamma}_{zz} \hat{k}_z \hat{k}'_z), \end{aligned} \quad (55)$$

where

$$V_1 = \frac{T_c}{2n} \left(\frac{\mu_B}{m} \right)^2 \frac{1}{\left(\frac{4\beta_z M_z^2}{\gamma k_F^2} + \frac{H_z}{4M_z \gamma k_F^2} + 1 \right)^2}. \quad (56)$$

Similarly for the off-diagonal components we have

$$V^{\uparrow\downarrow}(\mathbf{k}, \mathbf{k}') = -V_2 \tilde{\gamma}_{ij} \hat{k}_i \hat{k}'_j - iV_3 \tilde{\gamma}_{xy} (k_x k'_y + k'_x k_y), \quad (57)$$

$$V^{\downarrow\uparrow}(\mathbf{k}, \mathbf{k}') = -V_2 \tilde{\gamma}_{ij} \hat{k}_i \hat{k}'_j + iV_3 \tilde{\gamma}_{xy} (k_x k'_y + k'_x k_y), \quad (58)$$

where

$$V_2 = \frac{T_c}{2n} \left(\frac{\mu_B}{m} \right)^2 \left(\frac{1}{\left(\frac{1}{2\chi_x \gamma k_F^2} + 1 \right)^2} - \frac{1}{\left(\frac{1}{2\chi_y \gamma k_F^2} + 1 \right)^2} \right), \quad (59)$$

$$V_3 \cong \frac{T_c}{n} \left(\frac{\mu_B}{m} \right)^2 \frac{1}{\left(\frac{1}{\chi_x \gamma k_F^2} + 1 \right) \left(\frac{1}{\chi_y \gamma k_F^2} + 1 \right)}, \quad (60)$$

and $\tilde{\gamma}_{xy} = \gamma_{xy}/\gamma$. Due to relativistic smallness of coefficient $\tilde{\gamma}_{xy}$ in comparison with other coefficients $\tilde{\gamma}_{xx}, \dots$ the imaginary parts in Eqs. (57), (58) are always much smaller than its real parts.

One can see that the off-diagonal pairing amplitudes $V^{\uparrow\downarrow}$ and $V^{\downarrow\uparrow}$ are field independent, whereas the diagonal amplitudes $V^{\uparrow\uparrow}$ and $V^{\downarrow\downarrow}$ depend of magnetic field. Making use the estimations given by Eqs. (40), (41) for combination determining the denominator in Eq. (56) at $H_z = 0$ we have

$$\frac{4\beta_z M_z^2}{\gamma k_F^2} = 2 \frac{T_c(H_i) - T}{T_c}, \quad i = x, y. \quad (61)$$

Thus, at large enough magnetic field directed along either a or b axis this combination is small.

It is difficult to find explicitly the combinations $\frac{1}{2\chi_{x,y} \gamma k_F^2}$ determining the denominators in Eq. (59). In assumption that it is essentially larger than unity one can expect that

$$V^{\uparrow\uparrow} \gg V^{\uparrow\downarrow}, \quad (62)$$

otherwise these amplitudes are close in magnitude.

The explicit form of the pairing interaction is fixed and we can look now what kind of superconducting state it gives rise.

V. STRUCTURE OF SUPERCONDUCTING STATE

Even in the absence of external field in ferromagnetic superconductors there is an internal field acting on the

electron charges. Due to this reason the superconducting state is always inhomogeneous. So, working with Ginzburg-Landau-Gor'kov equations we should keep the gradient terms.

Performing the Taylor expansion of Eqs.(9), (10) in powers of \mathbf{q} up to the second order and then trans-

forming them to the coordinate representation, that means simple substitution

$$\mathbf{q} \rightarrow \mathbf{D} = -i\nabla_{\mathbf{r}} + 2e\mathbf{A}(\mathbf{r}), \quad (63)$$

we obtain equations

$$\begin{aligned} \Delta^\uparrow(\mathbf{k}, \mathbf{r}) = & T \sum_n \int \frac{d^3\mathbf{k}'}{(2\pi)^3} V_1 \tilde{\gamma}_{ij} \hat{k}_i \hat{k}'_j \left(G^\uparrow(\mathbf{k}', \omega_n) G^\uparrow(-\mathbf{k}', -\omega_n) + \frac{1}{2} G^\uparrow(\mathbf{k}', \omega_n) \frac{\partial^2 G^\uparrow(-\mathbf{k}', -\omega_n)}{\partial k'_i \partial k'_m} D_l D_m \right) \Delta^\uparrow(\mathbf{k}', \mathbf{r}) \\ & + T \sum_n \int \frac{d^3\mathbf{k}'}{(2\pi)^3} (V_2 \tilde{\gamma}_{ij} \hat{k}_i \hat{k}'_j + i V_3 \tilde{\gamma}_{xy} (k_x k'_y + k'_x k_y)) \left(G^\downarrow(\mathbf{k}', \omega_n) G^\downarrow(-\mathbf{k}', -\omega_n) + \frac{1}{2} G^\downarrow(\mathbf{k}', \omega_n) \frac{\partial^2 G^\downarrow(-\mathbf{k}', -\omega_n)}{\partial k'_i \partial k'_m} D_l D_m \right) \Delta^\downarrow(\mathbf{k}', \mathbf{r}) \quad (64) \\ \Delta^\downarrow(\mathbf{k}, \mathbf{r}) = & T \sum_n \int \frac{d^3\mathbf{k}'}{(2\pi)^3} (V_2 \tilde{\gamma}_{ij} \hat{k}_i \hat{k}'_j - i V_3 \tilde{\gamma}_{xy} (k_x k'_y + k'_x k_y)) \left(G^\uparrow(\mathbf{k}', \omega_n) G^\uparrow(-\mathbf{k}', -\omega_n) + \frac{1}{2} G^\uparrow(\mathbf{k}', \omega_n) \frac{\partial^2 G^\uparrow(-\mathbf{k}', -\omega_n)}{\partial k'_i \partial k'_m} D_l D_m \right) \Delta^\uparrow(\mathbf{k}', \mathbf{r}) \\ & + T \sum_n \int \frac{d^3\mathbf{k}'}{(2\pi)^3} V_1 \tilde{\gamma}_{ij} \hat{k}_i \hat{k}'_j \left(G^\downarrow(\mathbf{k}', \omega_n) G^\downarrow(-\mathbf{k}', -\omega_n) + \frac{1}{2} G^\downarrow(\mathbf{k}', \omega_n) \frac{\partial^2 G^\downarrow(-\mathbf{k}', -\omega_n)}{\partial k'_i \partial k'_m} D_l D_m \right) \Delta^\downarrow(\mathbf{k}', \mathbf{r}). \quad (65) \end{aligned}$$

Taking into account the angular structure of pairing interaction in orthorhombic ferromagnet given by equations (55), (57), (58) we can choose the superconducting order parameter as the following linear combinations of momentum direction projections on the coordinate axis

$$\Delta^\uparrow(\mathbf{k}, \mathbf{r}) = \hat{k}_x \eta_x^\uparrow(\mathbf{r}) + i \hat{k}_y \eta_y^\uparrow(\mathbf{r}) + \hat{k}_z \eta_z^\uparrow(\mathbf{r}), \quad (66)$$

$$\Delta^\downarrow(\mathbf{k}, \mathbf{r}) = \hat{k}_x \eta_x^\downarrow(\mathbf{r}) + i \hat{k}_y \eta_y^\downarrow(\mathbf{r}) + \hat{k}_z \eta_z^\downarrow(\mathbf{r}). \quad (67)$$

Substituting these expressions to Eqs. (65) and (65) we

come to the system of differential equations, that is convenient to present in the matrix form

$$\eta_\alpha(\mathbf{r}) = A_{\alpha\beta} \eta_\beta(\mathbf{r}) \quad (68)$$

for the components of vector

$$\eta_\alpha(\mathbf{r}) = (\eta_x^\uparrow(\mathbf{r}), \eta_x^\downarrow(\mathbf{r}), \eta_y^\uparrow(\mathbf{r}), \eta_y^\downarrow(\mathbf{r}), \eta_z^\uparrow(\mathbf{r}), \eta_z^\downarrow(\mathbf{r})) \quad (69)$$

and the matrix operator

$$A_{\alpha\beta} = \begin{pmatrix} g_{1x}^\uparrow \lambda + L_{1x}^\uparrow & g_{2x}^\downarrow \lambda + L_{2x}^\downarrow + i L_{3yx}^\downarrow & i L_{1xy}^\uparrow & -g_{3xy}^\downarrow \lambda + i L_{2xy}^\downarrow - L_{3y}^\downarrow & L_{1xz}^\uparrow & L_{2xz}^\downarrow + i L_{3yz}^\downarrow \\ g_{2x}^\uparrow \lambda + L_{2x}^\uparrow - i L_{3yx}^\uparrow & g_{1x}^\downarrow \lambda + L_{1x}^\downarrow & g_{3xy}^\uparrow \lambda + i L_{2xy}^\uparrow + L_{3y}^\uparrow & i L_{1xy}^\downarrow & L_{2zx}^\uparrow - i L_{3yz}^\uparrow & L_{1xz}^\downarrow \\ -i L_{1yx}^\uparrow & g_{3yx}^\downarrow \lambda - i L_{2yx}^\downarrow + L_{3x}^\downarrow & g_{1y}^\uparrow \lambda + L_{1y}^\uparrow & g_{2y}^\downarrow \lambda + L_{2y}^\downarrow + i L_{3xy}^\downarrow & -i L_{1yz}^\uparrow & -i L_{2yz}^\downarrow + L_{3xz}^\downarrow \\ -g_{3yx}^\uparrow \lambda - i L_{2yx}^\uparrow - L_{3x}^\uparrow & -i L_{1yx}^\downarrow & g_{2y}^\uparrow \lambda + L_{2y}^\uparrow - i L_{3xy}^\uparrow & g_{1y}^\downarrow \lambda + L_{1y}^\downarrow & -i L_{2yz}^\uparrow + L_{3xz}^\uparrow & -i L_{1yz}^\downarrow \\ L_{1zx}^\uparrow & L_{2zx}^\downarrow & i L_{1zy}^\uparrow & i L_{2zy}^\downarrow & g_{1z}^\uparrow \lambda + L_{1z}^\uparrow & g_{2z}^\downarrow \lambda + L_{2z}^\downarrow \\ L_{2zx}^\uparrow & L_{1zx}^\downarrow & i L_{2zy}^\uparrow & i L_{1zy}^\downarrow & g_{2z}^\uparrow \lambda + L_{2z}^\uparrow & g_{1z}^\downarrow \lambda + L_{1z}^\downarrow \end{pmatrix} \quad (70)$$

Here,

$$g_{1x}^\uparrow = V_1 \langle \tilde{\gamma}_{xx} \hat{k}_x^2 N_0^\uparrow(\hat{\mathbf{k}}) \rangle \quad (71)$$

is one of the constants of pairing interaction, the angular brackets mean the averaging over the Fermi sur-

face, $N_0^\uparrow(\hat{\mathbf{k}})$ is the angular dependent density of electronic states at the Fermi surface of the band \uparrow . Correspondingly

$$g_{2x}^\downarrow = V_2 \langle \tilde{\gamma}_{xx} \hat{k}_x^2 N_0^\downarrow(\hat{\mathbf{k}}) \rangle, \quad g_{3xy}^\downarrow = V_3 \langle \tilde{\gamma}_{xy} \hat{k}_y^2 N_0^\downarrow(\hat{\mathbf{k}}) \rangle. \quad (72)$$

All the other constants of pairing interaction are obtained by obvious substitutions $x \leftrightarrow y$ and $\uparrow \leftrightarrow \downarrow$ or $x \rightarrow z$.

The function

$$\lambda(T) = 2\pi T \sum_{n \geq 0} \frac{1}{\omega_n} = \ln \frac{\epsilon}{T}, \quad (73)$$

where $\epsilon = \frac{2\gamma\epsilon_0}{\pi}$, $\ln \gamma = 0.577$ is the Euler constant, and ϵ_0 is an energy cutoff for pairing interaction. We assume here that it has the same value for both bands.

The first type of differential operators is defined as follows

$$L_{1x}^\uparrow = \frac{1}{2} V_1 T \sum_n \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \tilde{\gamma}_{xx} \hat{k}_x^2 G^\uparrow(\mathbf{k}, \omega_n) \mathcal{D}, \quad (74)$$

and L_{2y}^\downarrow and the other operators with same structure are obtained by obvious substitutions $(x \rightarrow y, z)$, $(1 \rightarrow 2)$ and $(\uparrow \rightarrow \downarrow)$, but similar operator with index 3 is

$$L_{3x}^\uparrow = \frac{1}{2} V_3 T \sum_n \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \tilde{\gamma}_{xy} \hat{k}_y^2 G^\uparrow(\mathbf{k}, \omega_n) \mathcal{D}, \quad (75)$$

here,

$$\mathcal{D} = \frac{\partial^2 G^\uparrow(-\mathbf{k}, -\omega_n)}{\partial k_x^2} D_x^2 + \frac{\partial^2 G^\uparrow(-\mathbf{k}, -\omega_n)}{\partial k_y^2} D_y^2 + \frac{\partial^2 G^\uparrow(-\mathbf{k}, -\omega_n)}{\partial k_z^2} D_z^2. \quad (76)$$

The second type of operators is

$$L_{1xy}^\uparrow = \frac{1}{2} V_1 T \sum_n \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \tilde{\gamma}_{xx} \hat{k}_x \hat{k}_y G^\uparrow(\mathbf{k}, \omega_n) \frac{\partial^2 G^\uparrow(-\mathbf{k}, -\omega_n)}{\partial k_x \partial k_y} (D_x D_y + D_y D_x), \quad (77)$$

and L_{2yx}^\uparrow and the others operators of this type are obtained by obvious substitutions $(x \rightarrow y, z)$, $(1 \rightarrow 2)$, $(\uparrow \rightarrow \downarrow)$, but similar operators with index 3 are defined differently

$$L_{3xy}^\uparrow = \frac{1}{2} V_3 T \sum_n \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \tilde{\gamma}_{xy} \hat{k}_x \hat{k}_y G^\uparrow(\mathbf{k}, \omega_n) \frac{\partial^2 G^\uparrow(-\mathbf{k}, -\omega_n)}{\partial k_x \partial k_y} (D_x D_y + D_y D_x), \quad (78)$$

$$L_{3yz}^\uparrow = \frac{1}{2} V_3 T \sum_n \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \tilde{\gamma}_{xy} \hat{k}_y \hat{k}_z G^\uparrow(\mathbf{k}, \omega_n) \frac{\partial^2 G^\uparrow(-\mathbf{k}, -\omega_n)}{\partial k_y \partial k_z} (D_y D_z + D_z D_y). \quad (79)$$

A. Field along the spontaneous magnetization

For the arbitrary direction of external magnetic field we have 6 coupled equations for the 6 order parameter components. The situation is simplified in absence of external field, or when an external field is directed along the axis of spontaneous magnetization \hat{z} . Then the long derivatives are

$$D_x = -i \frac{\partial}{\partial x}, \quad D_z = -i \frac{\partial}{\partial z}, \quad D_y = -i \frac{\partial}{\partial y} + \frac{2e}{c} (H + H_{int}) x. \quad (80)$$

Here we have introduced internal electromagnetic field corresponding spontaneous magnetization $H_{int} \approx mn$ and ignore the difference between the external field and magnetic field induced inside the media by the external field.

Now the order parameter components are z -coordinate independent. Putting all the z derivative $D_z = 0$ we come to the matrix $A_{\alpha\beta}$ in the form

$$\begin{pmatrix}
g_{1x}^{\uparrow}\lambda + L_{1x}^{\uparrow} & g_{2x}^{\downarrow}\lambda + L_{2x}^{\downarrow} + iL_{3yx}^{\downarrow} & iL_{1xy}^{\uparrow} & -g_{3xy}^{\downarrow}\lambda + iL_{2xy}^{\downarrow} - L_{3y}^{\downarrow} & 0 & 0 \\
g_{2x}^{\uparrow}\lambda + L_{2x}^{\uparrow} - iL_{3yx}^{\uparrow} & g_{1x}^{\downarrow}\lambda + L_{1x}^{\downarrow} & g_{3xy}^{\uparrow}\lambda + iL_{2xy}^{\uparrow} + L_{3y}^{\uparrow} & iL_{1xy}^{\downarrow} & 0 & 0 \\
-iL_{1yx}^{\uparrow} & g_{3yx}^{\downarrow}\lambda - iL_{2yx}^{\downarrow} + L_{3x}^{\downarrow} & g_{1y}^{\uparrow}\lambda + L_{1y}^{\uparrow} & g_{2y}^{\downarrow}\lambda + L_{2y}^{\downarrow} + iL_{3xy}^{\downarrow} & 0 & 0 \\
-g_{3yx}^{\uparrow}\lambda - iL_{2yx}^{\uparrow} - L_{3x}^{\uparrow} & -iL_{1yx}^{\downarrow} & g_{2y}^{\uparrow}\lambda + L_{2y}^{\uparrow} - iL_{3xy}^{\uparrow} & g_{1y}^{\downarrow}\lambda + L_{1y}^{\downarrow} & 0 & 0 \\
0 & 0 & 0 & 0 & g_{1z}^{\uparrow}\lambda + L_{1z}^{\uparrow} & g_{2z}^{\downarrow}\lambda + L_{2z}^{\downarrow} \\
0 & 0 & 0 & 0 & g_{2z}^{\uparrow}\lambda + L_{2z}^{\uparrow} & g_{1z}^{\downarrow}\lambda + L_{1z}^{\downarrow}
\end{pmatrix} \quad (81)$$

We see that equations for

$$\Delta^{\uparrow}(\mathbf{k}, \mathbf{r}) = \hat{k}_x \eta_x^{\uparrow}(\mathbf{r}) + i\hat{k}_y \eta_y^{\uparrow}(\mathbf{r}), \quad (82)$$

$$\Delta^{\downarrow}(\mathbf{k}, \mathbf{r}) = \hat{k}_x \eta_x^{\downarrow}(\mathbf{r}) + i\hat{k}_y \eta_y^{\downarrow}(\mathbf{r}) \quad (83)$$

parts of the order parameter are decoupled from the equations for the components

$$\Delta^{\uparrow}(\mathbf{k}, \mathbf{r}) = \hat{k}_z \eta_z^{\uparrow}(\mathbf{r}), \quad (84)$$

$$\Delta^{\downarrow}(\mathbf{k}, \mathbf{r}) = \hat{k}_z \eta_z^{\downarrow}(\mathbf{r}), \quad (85)$$

as it should be because the terms $\hat{k}_x \eta_x^{\uparrow,\downarrow} + i\hat{k}_y \eta_y^{\uparrow,\downarrow}$ correspond to the order parameter transforming according to A_1 co-representation of the point symmetry group of orthorhombic ferromagnet, whereas the terms $\hat{k}_z \eta_z^{\uparrow,\downarrow}$ belong to B_1 co-representation.^{5,6}

The critical temperatures of transition to superconducting state A_1 or B_1 are defined by the maximum of eigenvalue of separate systems differential equations for corresponding components of the order parameter.

Using the relativistic smallness of $\tilde{\gamma}_{xy}$ one can neglect in matrix Eq.(81) by all the terms with index 3. This case the equations for the x components of the order parameter are coupled with the equations for the y components only due to the gradient terms like L_{1xy}^{\uparrow} etc. In absence of external field the gradient terms can be treated as small terms due to the smallness of internal electromagnetic field. Then the equations for x , y and z components of the order parameter are transformed into algebraic equations decoupled each other:

$$\begin{aligned}
\eta_x^{\uparrow} &= (g_{1x}^{\uparrow}\eta_x^{\uparrow} + g_{2x}^{\downarrow}\eta_x^{\downarrow})\Lambda, \\
\eta_x^{\downarrow} &= (g_{2x}^{\uparrow}\eta_x^{\uparrow} + g_{1x}^{\downarrow}\eta_x^{\downarrow})\Lambda,
\end{aligned} \quad (86)$$

$$\begin{aligned}
\eta_y^{\uparrow} &= (g_{1y}^{\uparrow}\eta_y^{\uparrow} + g_{2y}^{\downarrow}\eta_y^{\downarrow})\Lambda, \\
\eta_y^{\downarrow} &= (g_{2y}^{\uparrow}\eta_y^{\uparrow} + g_{1y}^{\downarrow}\eta_y^{\downarrow})\Lambda,
\end{aligned} \quad (87)$$

and

$$\begin{aligned}
\eta_z^{\uparrow} &= (g_{1z}^{\uparrow}\eta_z^{\uparrow} + g_{2z}^{\downarrow}\eta_z^{\downarrow})\Lambda, \\
\eta_z^{\downarrow} &= (g_{2z}^{\uparrow}\eta_z^{\uparrow} + g_{1z}^{\downarrow}\eta_z^{\downarrow})\Lambda.
\end{aligned} \quad (88)$$

Thus, in the exchange approximation for the energy of magnetic inhomogeneity we have three different superconducting states $(\hat{k}_x \eta_x^{\uparrow}, \hat{k}_x \eta_x^{\downarrow})$, $(\hat{k}_y \eta_y^{\uparrow}, \hat{k}_y \eta_y^{\downarrow})$ and $(\hat{k}_z \eta_z^{\uparrow}, \hat{k}_z \eta_z^{\downarrow})$ with different critical temperatures defined by the determinants of Eqs. (86), (87) and (88).

VI. FIELD DEPENDENT CRITICAL TEMPERATURE

Let us discuss the situation in neglect the orbital effects that is as if the magnetic field acts only on the electron spins.

Assuming that the largest critical temperature corresponds to the $(\hat{k}_x \eta_x^{\uparrow}, \hat{k}_x \eta_x^{\downarrow})$ superconducting state the zero of determinant of the system (86) yields the BSC-type formula

$$T = \epsilon \exp\left(-\frac{1}{g}\right), \quad (89)$$

where the constant of interaction

$$g = \frac{g_{1x}^{\uparrow} + g_{1x}^{\downarrow}}{2} + \sqrt{\frac{(g_{1x}^{\uparrow} - g_{1x}^{\downarrow})^2}{4} + g_{2x}^{\uparrow}g_{2x}^{\downarrow}} \quad (90)$$

is the function of temperature and magnetic field. The formula (89) is, in fact the equation for the determination of the critical temperature of the transition to the superconducting state $T_{sc} = T_{sc}(\mathbf{H})$.

As we have seen in the Chapter IV the constants g_{1x}^{\uparrow} and g_{1x}^{\downarrow} are field dependent and the constants g_{2x}^{\uparrow} and g_{2x}^{\downarrow} are not. The Curie temperature in URhGe at zero field is about 10 K, and it is practically field independent in magnetic fields smaller than the upper critical field both along a and b directions²² whereas the superconductivity is developed at temperatures below $T_{sc} \approx 0.25$ K. It means that the combination in equation (61) $\frac{4\beta_z M_z^2}{\gamma k_F^2} \approx 2$ and all the pairing amplitudes are proved to be field independent. Thus the critical temperature of superconducting phase transition is independent of magnetic field.

Another situation is realized in UCoGe in field parallel to b -axis. It is impossible to say something definite about

the absolute value of the pairing constants. However, one can easily compare their relative values in different field regions. It is simplest to consider this in the case of validity of inequality (62) that corresponds to single band superconductivity. Then the critical temperature in field parallel to b -axis is determined by

$$T = \epsilon \exp\left(-\frac{1}{g_{1x}^\uparrow}\right), \quad (91)$$

where according to Eqs. (56) and (61)

$$g_{1x}^\uparrow \propto \frac{1}{(2\frac{T_c(H_y)-T}{T_c} + 1)^2}. \quad (92)$$

In zero magnetic field $T_c = 3.5$ K and $T_{sc} = 0.55$ K, hence, in small enough magnetic fields, so long the Curie temperature is field independent, $\frac{T_c(H_y)-T}{T_c} \approx 1$ and the critical temperature of superconducting transition is field independent as it is in URhGe. However, for field $H_y > 5$ Tesla the Curie temperature begins sharp decreasing, the combination $\frac{T_c(H_y)-T}{T_c}$ starts to be less and less than 1. This leads to the sharp increase of the pairing constant g_x^\uparrow and the effective superconducting critical temperature grows exponentially, that corresponds to the robustness of superconducting state in field interval $5 < H_y < 10$ Tesla.¹¹

Similar mechanism also stimulates the arising the reentrant superconducting state in URhGe. Indeed, the Curie temperature in fields along b -axis about 10 Tesla is significantly smaller than it is in zero field.²⁰ This leads to drastic increase of pairing interaction and reappearance of the superconducting state reported in Ref.13. One must stress that the reentrant superconducting state appears already at field about 8 Tesla that is in significantly lower field H_b than the field of the first order transition equal to 12 Tesla reported in papers^{23,24}.

The magnetic anisotropy in a direction is much harder and a decrease of the Curie temperature in fields H_x up to 10-12 Tesla has not yet reported. However, at low temperatures when the upper critical field along a direction is larger than 10 Tesla the upper critical field in UCoGe reveals the upward curvature. This in our opinion corresponds to the beginning of suppression of the Curie temperature and corresponding increase of the pairing interaction like it is in smaller fields directed along b axis.

The above discussion is applicable to the situations with the external field parallel either b or a axis. In pres-

ence also the field H_z along the direction of spontaneous magnetization one must work with pairing constant g_{1x}^\uparrow proportional to pairing potential V_1 given by general formula (56) where M_z is determined either by Eq. (27) or by Eq. (29). One can see that the field along c axis increases the magnetic moment and definitely decreases the pairing amplitude g_{1x}^\uparrow . This is probably the source of non GL sharp anisotropy of the upper critical field in UCoGe observed¹¹ at very low temperature $T=0.85$ K at quite small field inclinations from a -axis direction toward c axis.

VII. CONCLUSION

We have derived the relationship between the magnetic and superconducting properties in uranium ferromagnetic superconductors. It was shown that the long time ago established general form of the superconducting order parameter dictated by symmetry^{5,6} directly follows from the microscopic model where the interaction between the electrons is supported by the magnetization fluctuations in ferromagnetic metal with orthorhombic symmetry.

The suppression of the Curie temperature by the magnetic field in b -direction results in the strong robustness of superconducting state in UCoGe at fields exceeding ≈ 5 Tesla. The same mechanism is probably responsible for the low temperature upturn of the upper critical field parallel to a -axis in the same material reported in Ref.11.

The developed theory allows to explain several basic properties of uranium ferromagnetic superconductors like the order parameter structure, presence of nodes in quasi-particle spectrum and temperature dependence of upper critical field. Certainly, there are some properties of these materials like field dependent effective mass³ and strong anisotropy of NMR and NQR attenuation¹² which could find an explanation in more elaborate strong coupling theory. However, even our simplified treatment demonstrate that due to multi-band and multi-component character of superconductivity to reach full quantitative description of superconducting state in these materials is by no means a simple problem.

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